FREQUENCY DOMAIN ERRORS-IN-VARIABLES APPROACH FOR TWO-CHANNEL SIMO SYSTEM IDENTIFICATION

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ABSTRACT

This paper deals with the blind identification of the variances of the additive noises in a two-channel SIMO system. They are estimated by using a frequency domain errors-in-variables scheme, when finite impulse responses (FIR) filters are driven by a white input sequence. It should be noted that the proposed approach has the advantage of addressing the unbalanced case, i.e. when the noise variances are not the same. Moreover, the algorithm enables the solution to be directly obtained from the noisy data and no optimization algorithm is required.

1. INTRODUCTION

For the last years, a great deal of interest has been paid to single input multiple output (SIMO) system identification [1]–[5] in signal processing. Thus, in the framework of speech enhancement based on two microphones, blind identification methods such as [6] have been proposed to retrieve the speech signal from reverberated observations disturbed by additive noises. In mobile communications, identification approaches have also been of interest to carry out equalization of the received signal [7].

In the following, let us consider the SIMO system depicted in figure 1, where $s(k)$ denotes the input signal of $M = 2$ filters, defined by their finite impulse responses (FIRs) $h_i(k)$, $(i = 1, 2)$, with maximal order L. The filter outputs $x_i(k)$ are then disturbed by additive zero-mean noises $b_i(k)$, with variance σ_i^2, $(i = 1, 2)$. This yields the noisy observations $y_i(k)$, $(i = 1, 2)$. Moreover, $s(k)$ is assumed to be a stationary zero-mean white process with unit–variance, without loss of generality.
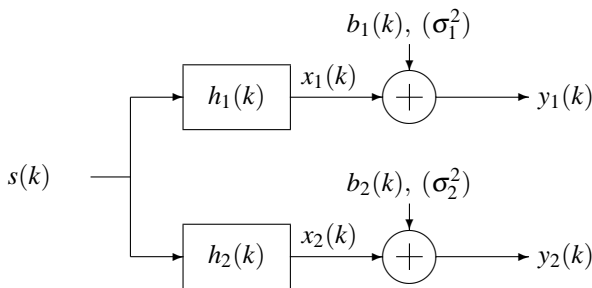

Figure 1: two-channel SIMO system

SIMO system identification consists in estimating the FIRs and may include the estimations of the variances of the additive noises[1]. Thus, Xu et al. [1] exploit the so–called

cross–relations between the filtered outputs:

$$
\begin{aligned}
h_j(k) * x_i(k) &= h_j(k) * \big(h_i(k) * s(k)\big) \\
&= \big(h_i(k) * h_j(k)\big) * s(k) = h_i(k) * x_j(k).
\end{aligned} \tag{1}
$$

For this purpose, they search the null space of a suitable positive hermitian block Toeplitz matrix, built with the cross–correlation coefficients of the noisy filter outputs. As an alternative, the subspace based method presented in [2] consists in retrieving the span of the autocorrelation matrix R_L of the vector $\underline{x}(k) = \big[\underline{x}_1^T(k) \cdots \underline{x}_M^T(k)\big]^T$ with $\underline{x}_i(k) = [x_i(k) \cdots x_i(k-L)]^T$. It should be noted that both methods are equivalent when dealing with two channels.

Adaptive approaches have also been studied, either in the time domain [3] or in the frequency domain [4] [5]. They aim at minimizing the cross-relations error, i.e. $E\left[|h_j(k) * y_i(k) - h_i(k) * y_j(k)|^2\right]$ and are based on the least mean square (LMS) and Newton algorithms applied to the noisy data.

However, the above methods cannot be used when the additive noises do not have the same variance, i.e. when dealing with the "unbalanced" case. Few methods deal with this issue. In [6] and [8], the approach is based on the Frisch scheme [9], which is a class of errors–in–variables (EIV) solutions. The EIV models [11], initially developed in the fields of statistics and identification, assume that the available data are disturbed by additive error terms. Given a generic process described by K variables $\{t_k\}_{k=1,\cdots,K}$, the formulation of an EIV estimation problem consists in determining, on the only basis of noisy observations $\{z_k = t_k + n_k\}_{k=1,\cdots,K}$, the set of K-tuple $\{\lambda_k\}_{k=1,\cdots,K}$ that satisfies:

$$
\lambda_1 t_1 + \lambda_2 t_2 + \cdots + \lambda_K t_K = 0
$$

or equivalently,

$$
\mathbf{R_t}^T \begin{bmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_K \end{bmatrix}^T = 0 \tag{2}
$$

where $\mathbf{R_t}$ is the covariance matrix of $\{t_k\}_{k=1,\cdots,K}$ and the upperscript ^{T} denotes the transpose.

When each noise term n_k is assumed to be zero-mean, independent of every other noise term and every variable t_k, one has:

$$
\mathbf{R_t} = \mathbf{R_z} - \mathbf{R_n}
$$

where $\mathbf{R_z}$ and $\mathbf{R_n}$ respectively denote the covariance matrix of $\{z_k\}_{k=1,\cdots,K}$ and of $\{n_k\}_{k=1,\cdots,K}$.

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[1]It should be noted that, in some specific cases, the variance of each additive noise could be estimated independently of each other. In speech processing, it could be updated during silent frames. Subspace methods [10] could also be considered provided that the spectrum of the noise-free signal is discrete. However, these cases are too restrictive and we propose here to study a more general solution.

At that stage, the so-called Frisch scheme [9] consists in searching for the diagonal matrices $\mathbf{R_n}$ which enable $\mathbf{R_z} - \mathbf{R_n}$ to be positive semidefinite:

$$\mathbf{R_z} - \mathbf{R_n} \geq 0 \qquad (3)$$

When dealing with SIMO system identification, given the cross relations (1) and by selecting $K = 2L+2$, the set of $2L+2$ variables $\{t_k\}_{k=1,\cdots,2L+2}$ can be for instance defined as follows:

$$[t_1 \cdots t_{2L+2}]^T = [x_2(k) \cdots x_2(k-L) \ -x_1(k) \cdots -x_1(k-L)]^T,$$

whereas the $(2L+2)$-tuple $(\lambda_1, \cdots, \lambda_{2L+2})$ satisfies:

$$[\lambda_1 \cdots \lambda_{2L+2}]^T = [h_1(0) \cdots h_1(L) \ h_2(0) \cdots h_2(L)]^T.$$

In that case, z_k corresponds to the noisy observations. According to [8], solving (3) yields an infinite set S_L of variances (σ_1^2, σ_2^2). In [6], we suggest carrying out once again the Frisch scheme by selecting $K = 2l+2$ with $l > L$ to find the true variances which belong to $S_L \cap S_l$. Then, the impulse responses can be retrieved according to (2). However, there are two main drawbacks:

1. There may be spurious solutions if l is not suitably chosen ($l - L$ must not be too low).
2. We do not take benefit of the fact that the true variances belong to every set S_l with $l \geq L$. Indeed, for the sake of simplicity and computation, only two Frisch schemes are carried out.

Therefore, in this paper, we propose to develop a frequency domain EIV approach for SIMO identification. This method avoids the above drawbacks. The rest of the paper is organized as follows: section 2.1 deals with the variance estimation issue. The identifiability conditions and the proposed algorithms are introduced in section 2.3. Then, experimental results are presented in section 3.

2. SIMO SYSTEM IDENTIFICATION IN THE FREQUENCY DOMAIN

2.1 Focus on the variance estimation issue

Given the assumptions on the input signal $s(k)$ made in the introduction above, the power spectrum density matrix of the vector $\underline{x}(k) = [x_2(k) \ -x_1(k)]^T$ is defined by:

$$\begin{aligned} X(w) &:= \begin{bmatrix} X_{11}(w) & X_{12}(w) \\ X_{21}(w) & X_{22}(w) \end{bmatrix} \\ &= \begin{bmatrix} |H_2(w)|^2 & -H_2(w)H_1^*(w) \\ -H_1^*(w)H_2(w) & |H_1(w)|^2 \end{bmatrix}. \end{aligned}$$

$X(w)$ is positive. In addition, it is singular since one has

$$\det\big(X(w)\big) = 0.$$

Both properties are therefore denoted as follows:

$$X(w) \geq 0. \qquad (4)$$

However, only the power spectrum density $Y(w)$ of the noisy observation vector $\underline{y}(k) = [y_2(k) \ -y_1(k)]^T$ is available. According to figure 1, it satisfies:

$$\begin{aligned} Y(w) &:= \begin{bmatrix} Y_{11}(w) & Y_{12}(w) \\ Y_{21}(w) & Y_{22}(w) \end{bmatrix} = X(w) + B(w) \\ &= X(w) + \begin{bmatrix} \sigma_2^2 & 0 \\ 0 & \sigma_1^2 \end{bmatrix}. \end{aligned} \qquad (5)$$

Taking into account (4) and (5), the noise variances are solutions of the following equation system:

$$X_{\alpha_1, \alpha_2}(w) := \det\left(Y(w) - \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \right) = 0 \qquad (6)$$

$$\text{trace}\left(Y(w) - \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \right) \geq 0 \quad (w \in [-\pi; \pi]) \qquad (7)$$

where α_1 and α_2 are the unknowns.

Estimating the noise variances is very useful as it makes it possible to estimate $X(w)$ and consequently to retrieve the impulse responses $h_i(k)$ ($i = 1, 2$) since equation (2), or equivalently equation (1), can be rewritten in the frequency domain as follows:

$$\text{Ker}\big(X^H(w)\big) = \text{Span} \begin{bmatrix} H_1(w) \\ H_2(w) \end{bmatrix}$$

In the next two subsections, we will study how to solve the system (6)-(7), and will present algorithms for the noise variance estimation.

2.2 First approach to blindly estimate the variances

In addition to the trivial solution (σ_1^2, σ_2^2), the system (6) and (7) admits another solution if and only if one can find $(\gamma, \rho) \in \mathbb{R}_+^* \times \mathbb{R}^*$ such as

$$X_{11}(w) - \gamma X_{22}(w) = \rho. \qquad (8)$$

See appendix 1 for proof.

Then, a first approach we could consider to blindly estimate the noise variances would consist in minimizing the following cost function:

$$J_1(\alpha_1, \alpha_2) = \left\| X_{\alpha_1, \alpha_2}(w) \right\|^2 = \left\langle X_{\alpha_1, \alpha_2}(w) \middle| X_{\alpha_1, \alpha_2}(w) \right\rangle \qquad (9)$$

where $< .|. >$ denotes the hermitian product defined as follows:

$$\left\langle A(w) \middle| B(w) \right\rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} A(w) B^*(w) \mathrm{d}w. \qquad (10)$$

The hermitian product (10) can be expressed by using the Fourier coefficients of the 2π-periodic functions $A(w)$ and $B(w)$, denoted a^k and b^k respectively. Indeed, since $A(w) = \sum_{k \in \mathbb{Z}} a^k e^{jkw}$ and $B(w) = \sum_{k \in \mathbb{Z}} b^k e^{jkw}$, one obtains

$$\begin{aligned} \left\langle A(w) \middle| B(w) \right\rangle &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\sum_{k \in \mathbb{Z}} a^k e^{jkw} \right) \left(\sum_{k \in \mathbb{Z}} b^k e^{jkw} \right)^* \mathrm{d}w \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \sum_{n \in \mathbb{Z}} \left[\sum_{k \in \mathbb{Z}} a^k \left(b^{k-n} \right)^* \right] e^{jnw} \mathrm{d}w \\ &= \sum_{k \in \mathbb{Z}} a^k \left(b^k \right)^*. \end{aligned}$$

Therefore, when dealing with hermitian products between the elements of the power spectrum density matrices such as $X_{ij}(w)$ for instance $(i, j = 1, 2)$, the hermitian product can be hence defined from the intercorrelation sequences of $x_i(k)$ and $x_j(k)$.

The gradient of $J_1(\alpha_1, \alpha_2)$ is given by

$$\frac{\partial J(\alpha_1, \alpha_2)}{\partial \alpha_i} = 2 \left\langle \frac{\partial X_{\alpha_1, \alpha_2}(w)}{\partial \alpha_i} \middle| X_{\alpha_1, \alpha_2}(w) \right\rangle, \ (i = 1, 2),$$

When cancelling it, the following two equations have to be solved:

$$\alpha_1 = \phi_2(\alpha_2) \tag{11}$$

$$:= \frac{y_{11}^0 \alpha_2^2 - \left(g^0 + \langle Y_{22}(w)|Y_{11}(w)\rangle\right)\alpha_2 + \langle Y_{22}(w)|G(w)\rangle}{\alpha_2^2 - 2y_{22}^0\alpha_2 + \|Y_{22}(w)\|^2}$$

$$\alpha_2 = \phi_1(\alpha_1) \tag{12}$$

$$:= \frac{y_{22}^0 \alpha_1^2 - \left(g^0 + \langle Y_{11}(w)|Y_{22}(w)\rangle\right)\alpha_1 + \langle Y_{11}(w)|G(w)\rangle}{\alpha_1^2 - 2y_{11}^0\alpha_1 + \|Y_{11}(w)\|^2}.$$

The solution of (11) end (12) are given by searching the roots of a 5th degree real polynomial, depending on either α_1 or α_2.

In theory, the variances of the additive noises can be obtained as follows:

- when the trivial solution (σ_2^2, σ_1^2) is the only solution to the equation (6), it can be obtained by minimizing the cost function $J_1(\alpha_1, \alpha_2)$.
- When there are two solutions to the equation (6), that can be also found by minimizing $J_1(\alpha_1, \alpha_2)$, there exists $\gamma \in \mathbb{R}^*$ and $\rho \in \mathbb{R}^*$ such as

$$X_{11}(w) - \gamma X_{22}(w) = \rho. \tag{13}$$

See appendix 2. However, when $\gamma > 0$, only the trivial solution satisfies (7).

Nevertheless, in real cases and when (6) admits two solutions, the cost function $J_1(\alpha_1, \alpha_2)$ can then have very small values in the neighborhood of theses solutions. As a consequence, retrieving them becomes a hard task, due to the errors introduced when computing $Y(w)$ from a finite number of available samples:

- on the one hand, $B(w)$ is not diagonal.
- On the other hand, the unbiased estimates of the intercorrelation sequences are considered.

To illustrate this point, let us consider the example given below where the two following FIRs satisfy:

$$h_1 = \left\{2, 1, -1, 2\right\}$$

$$h_2 = \left\{\frac{4}{3}, \frac{-65 + \sqrt{5521}}{54}, \frac{65 - \sqrt{5521}}{24}, 3\right\},$$

In figure 2 the theoritical cost function $J_1(\alpha_1, \alpha_2)$ plotted in dB exhibits the trivial solution and the spurious one. Nevertheless, it takes very small values in the neighborhood of both solutions. In figure 3, when a real simulation is completed with $N = 10000$ observations samples of y_1 and y_2, the estimated cost function $J_1(\alpha_1, \alpha_2)$ does not make it possible to point out the true solution.

More generally, we will see that this problem happens when

$$\left|\langle \widetilde{X}_{11}(w)|\widetilde{X}_{22}(w)\rangle\right| = \left\|\widetilde{X}_{11}(w)\right\|\left\|\widetilde{X}_{22}(w)\right\|$$

where $\widetilde{X}_{11}(w)$ abd $\widetilde{X}_{22}(w)$ are defined as follows:

$$X_{11}(w) = \widetilde{X}_{11}(w) + x_{11}^0 \tag{14}$$

$$X_{22}(w) = \widetilde{X}_{22}(w) + x_{22}^0, \tag{15}$$

with $x_{11}^0 = <X_{11}^0(w)|1>$ and $x_{22}^0 = <X_{22}^0(w)|1>$ denoting the mean values of $X_{11}(w)$ and $X_{11}(w)$ respectively. This will be confirmed in the next subsection.

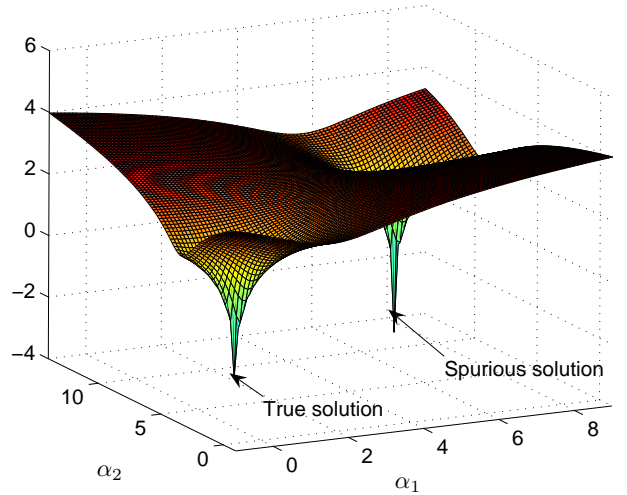Figure 2: theoritical criterion $J_1(\alpha_1, \alpha_2)$ when a spurious solution to (6) exists. Here, $\gamma > 0$.
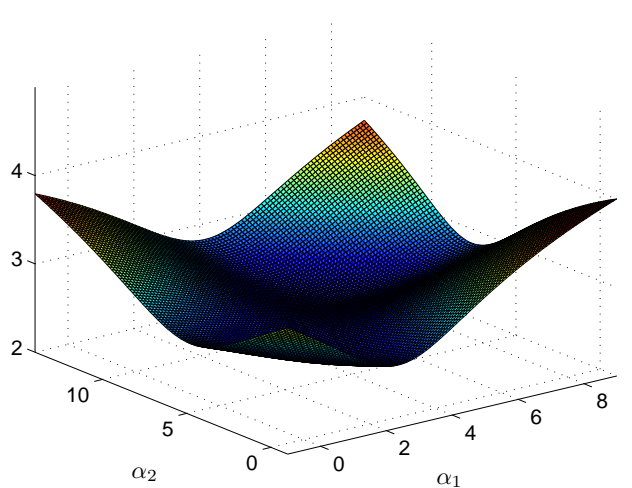

Figure 3: criterion $J_1(\alpha_1, \alpha_2)$ in dB, in a real case while a spurious solution to (6) should have existed. $N = 10000$.

2.3 Second approach to blindly estimate the noise variances

We propose a new criterion to estimate the noise variances when the equation (6) has a single solution. It is based on the following property. Let us assume that both $\widetilde{X}_{11}(w) \neq 0$ and $\widetilde{X}_{22}(w) \neq 0$, and $X_{11}(w) \neq X_{22}$. According to appendix 2, we have:

$(a) \qquad X_{\alpha_1, \alpha_2}(w) = 0 \quad$ has a single solution $\quad \Leftrightarrow$

$(b) \qquad \widetilde{X}_{\alpha_1, \alpha_2}(w) = 0 \quad$ has a single solution $\quad \Leftrightarrow$

$(c) \qquad \left|\langle \widetilde{X}_{11}(w)|\widetilde{X}_{22}(w)\rangle\right| < \left\|\widetilde{X}_{11}(w)\right\|\left\|\widetilde{X}_{22}(w)\right\| \tag{16}$

where $\widetilde{X}_{\alpha_1, \alpha_2}(w)$ is defined as in (14)-(15).

Then, when there is a single solution to equation (6), it can

be found by solving equation

$$\widetilde{X}_{\alpha_1,\alpha_2}(w) = 0.$$

Therefore, we propose to minimize the following criterion:

$$
\begin{aligned}
J_2(\alpha_1,\alpha_2) &= \left\| \widetilde{X}_{\alpha_1,\alpha_2}(w) \right\|^2 \\
&= \left\| \widetilde{G}(w) - \alpha_1 \widetilde{Y}_{22}(w) - \alpha_2 \widetilde{Y}_{11}(w) + \alpha_1 \alpha_2 \right\|^2 \quad (17)
\end{aligned}
$$

where $G(w) = Y_{11}(w)Y_{22}(w) - Y_{21}(w)Y_{12}(w)$ and $\widetilde{G}(w)$ is defined as in (14)-(15).
$J_2(\alpha_1,\alpha_2)$ is a quadratic function. Equating to zero the gradient of $J_2(\alpha_1,\alpha_2)$ leads to the following equations:

$$
\left\langle -\widetilde{Y}_{22}(w) \;\middle|\; \widetilde{G}(w) - \alpha_2 \widetilde{Y}_{11}(w) - \alpha_1 \widetilde{Y}_{22}(w) \right\rangle = 0
$$

$$
\left\langle -\widetilde{Y}_{11}(w) \;\middle|\; \widetilde{G}(w) - \alpha_1 \widetilde{Y}_{22}(w) - \alpha_2 \widetilde{Y}_{11}(w) \right\rangle = 0
$$

One finally deduces:

$$
\alpha_1 = \frac{-\langle \widetilde{Y}_{22}(w)|\widetilde{Y}_{11}(w)\rangle \alpha_2 + \langle \widetilde{Y}_{22}(w)|\widetilde{G}(w)\rangle}{\|\widetilde{Y}_{22}(w)\|^2} \quad (18)
$$

$$
\alpha_2 = \frac{-\langle \widetilde{Y}_{22}(w)|\widetilde{Y}_{11}(w)\rangle \alpha_1 + \langle \widetilde{Y}_{11}(w)|\widetilde{G}(w)\rangle}{\|\widetilde{Y}_{11}\|^2} \quad (19)
$$

Hence, this linear system can be re-written as follows:

$$
\underbrace{\begin{bmatrix} 1 & \dfrac{\langle \widetilde{Y}_{22}(w)|\widetilde{Y}_{11}(w)\rangle}{\|\widetilde{Y}_{22}(w)\|^2} \\ \dfrac{\langle \widetilde{Y}_{22}(w)|\widetilde{Y}_{11}(w)\rangle}{\|\widetilde{Y}_{11}(w)\|^2} & 1 \end{bmatrix}}_{M} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\langle \widetilde{Y}_{22}(w)|\widetilde{G}(w)\rangle}{\|\widetilde{Y}_{22}(w)\|^2} \\ \dfrac{\langle \widetilde{Y}_{11}(w)|\widetilde{G}(w)\rangle}{\|\widetilde{Y}_{11}(w)\|^2} \end{bmatrix}
$$

One retrieves the condition (16)-(c) by imposing $\det(M) > 0$.

It should be noted that the condition (7) is no longer necessary when equation (6) is assumed to have a unique solution.

3. RESULTS

We propose to illustrate the algorithm with two examples.

The first example deals with a $L = 2^{nd}$ order system, the FIRs corresponding to the following transfer functions are defined by:

$$H_1(z) = 1 + z + z^{-1} \quad \text{and} \quad H_2(z) = 1 - z + z^{-1}.$$

$N = 150$ samples are assumed to be available. Moreover, the two curves in the plan (α_1,α_2) defined by the relations (11) and (12) for the first algorithm, and the two lines in the plane (α_1,α_2) defined by the relations (18) and (19) for the second algorithm, are represented in figure 4.

In the second example, the proposed algorithms are then tested with higher-order systems ($L = 100$), whose time and frequential domain representations are given in figure 5. Here, $N = 1000$ samples are available. The good performances are illustrated in figure 6.

Numerical results based on Monte Carlo simulations are given in Table 1 and 2, respectively for the examples 1 and 2. In each case, the signal–to–noise ratios (*SNR*) for each channel are respectively equal to $SNR_1 = 10dB$ and $SNR_2 = 5dB$. The estimations of the variances provided by the cost functions $J_1(\alpha_1,\alpha_2)$ (9) and $J_2(\alpha_1,\alpha_2)$ (17) are compared. In every case, only few samples (i.e. a few hundred) are required to estimate the pair (σ_1^2,σ_2^2).
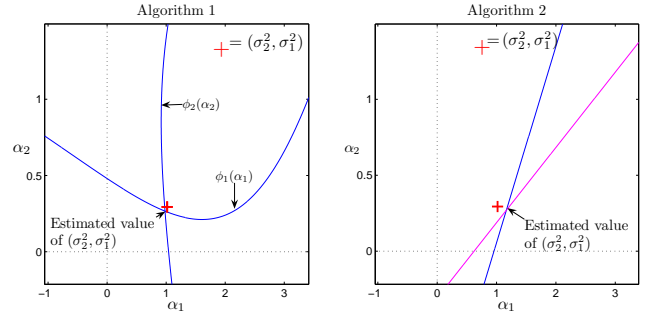

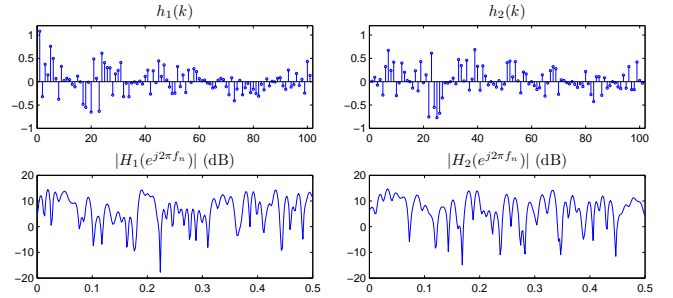Figure 4: noise variances estimation for example 1.

Figure 5: L^{th} order FIR and their spectral representations.
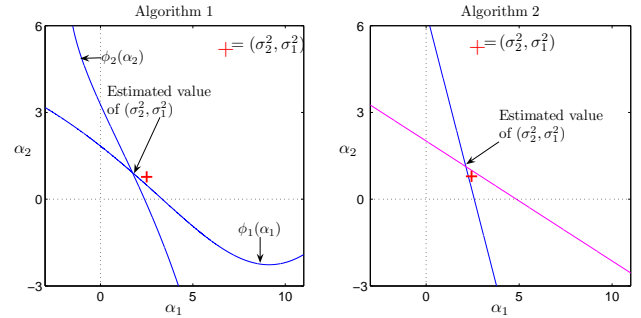

Figure 6: noise variances estimation for example 2.

Table 1: variance estimation and standard deviation based on 100 noise realizations, for the SIMO system of example 1.

example 1	$\sigma_1^2 = 0.290$	$\sigma_2^2 = 0.998$
Criterion $J_1(\alpha_1,\alpha2)$	0.383 ± 0.054	0.930 ± 0.100
Criterion $J_2(\alpha_1,\alpha2)$	0.498 ± 0.121	1.042 ± 0.173

Table 2: variance estimation and standard deviation based on 100 noise realizations, for the SIMO system of example 2.

example 2	$\sigma_1^2 = 0.776$	$\sigma_2^2 = 2.318$
Criterion $J_1(\alpha_1,\alpha2)$	1.252 ± 0.357	1.836 ± 0.338
Criterion $J_2(\alpha_1,\alpha2)$	1.506 ± 0.447	2.188 ± 0.427

4. CONCLUSION

In this paper, the variance estimation issue in the unbalanced case is addressed. For this purpose, we propose a frequency

domain errors-in-variables approach. More particularly, we investigate a first criterion and its the limits and hence propose an alternative one. This criterion has the advantage of being simpler than the previous one since it is quadratic.

5. APPENDIX 1

Let us assume that $(\alpha_1, \alpha_2) \neq (\sigma_2^2, \sigma_1^2)$ is a solution of (6). Then, $\alpha_1 \neq \sigma_2^2$ AND $\alpha_2 \neq \sigma_1^2$. By rewritting (6) as follows:

$$\Big(X_{11}(w) + \sigma_2^2 - \alpha_1\Big)\Big(X_{22}(w) + \sigma_1^2 - \alpha_2\Big) = X_{11}(w)X_{22}(w),$$

there is necessarily $\gamma \in \mathbb{R}^*$ such that:

$$X_{11}(w) + \sigma_2^2 - \alpha_1 = \gamma X_{22}(w) \tag{20}$$

$$X_{22}(w) + \sigma_1^2 - \alpha_2 = \frac{1}{\gamma} X_{11}(w). \tag{21}$$

Moreover, since the pair (α_1, α_2) also satisfies (7), one deduces:

$$X_{11}(w) + X_{22}(w) + \sigma_2^2 - \alpha_1 + \sigma_1^2 - \alpha_2$$
$$= \gamma X_{22}(w) + \frac{1}{\gamma} X_{11}(w) \geq 0$$

Necessarily, $\gamma > 0$. By introducing $\rho = \alpha_1 - \sigma_2^2$ and in the relation (20), one obtains (8).

Now, one assumes that $X_{11}(w) - \gamma X_{22}(w) = \rho$ with $\gamma > 0$ and $\rho \in \mathbb{R}^*$. Then, the pair $(\alpha_1, \alpha_2) = \rho + \sigma_2^2, \sigma_1^2 - \frac{\rho}{\gamma}$ is a non-trivial solution of (6). Moreover, one can write:

$$\text{trace}\left(X(w) + \begin{bmatrix} -\rho & 0 \\ 0 & \frac{\rho}{\gamma} \end{bmatrix}\right) = \Big(X_{11}(w) - \rho\Big) + \Big(X_{22}(w) + \frac{\rho}{\gamma}\Big)$$

According to relation (8) and since both $\gamma > 0$ and the functions $X_{11}(w)$ and $X_{22}(w)$ are positive, one has:

$$X_{11}(w) - \rho = \gamma X_{22}(w) \geq 0$$
$$X_{22}(w) + \frac{\rho}{\gamma} = \frac{1}{\gamma} X_{11}(w) \geq 0$$

Finally, one obtains

$$\text{trace}\left(X(w) + \begin{bmatrix} -\rho & 0 \\ 0 & \frac{\rho}{\gamma} \end{bmatrix}\right) \geq 0.$$

So, $(\rho + \sigma_2^2, \sigma_1^2 - \frac{\rho}{\gamma})$ satisfies the relation (7).

6. APPENDIX 2

Proof: Let us first expand the equation (6) as follows:

$$X_{\alpha_1, \alpha_2}(w) = 0 \qquad \Leftrightarrow$$
$$\begin{cases} (i) & \widetilde{X}_{\alpha_1, \alpha_2}(w) = 0 \\ (ii) & (\sigma_1^2 - \alpha_2)x_{11}^0 + (\sigma_2^2 - \alpha_1)x_{22}^0 \\ & + (\sigma_1^2 - \alpha_2)(\sigma_2^2 - \alpha_1) = 0. \end{cases} \tag{22}$$

where $\widetilde{X}_{\alpha_1, \alpha_2}(w)$ can be developped as follows:

$$\widetilde{X}_{\alpha_1, \alpha_2}(w) = (\sigma_1^2 - \alpha_2)\widetilde{X}_{11}(w) + (\sigma_2^2 - \alpha_1)\widetilde{X}_{22}(w).$$

Hence, $(b) \Rightarrow (a)$. To prove the converse part, let us assume a non-zero solution to (22)-(i). Then $\alpha_1 \neq \sigma_2^2$ AND $\alpha_2 \neq \sigma_1^2$

since both $\widetilde{X}_{11}(w) \neq 0$ and $\widetilde{X}_{22}(w) \neq 0$. As a consequence, there is $\gamma \in \mathbb{R}^*$ such as:

$$(\sigma_2^2 - \alpha_1) = -\gamma(\sigma_1^2 - \alpha_2) \tag{23}$$
$$\widetilde{X}_{11} = \gamma \widetilde{X}_{22}. \tag{24}$$

Substituing (23) and (24) in (22)-(ii) yields the following relation:

$$\alpha_1 = \sigma_2^2 - \gamma x_{22}^0 + x_{11}^0 \tag{25}$$
$$\alpha_2 = \sigma_1^2 - \frac{x_{11}^0}{\gamma} + x_{22}^0 \tag{26}$$

Finally, since $X_{11}(w) \neq X_{22}$, $-\gamma x_{22}^0 + x_{11}^0 \neq 0$, the pair (α_1, α_2) defined by the relations (25) and (26) satisfies $\alpha_1 \neq \sigma_2^2$ and $\alpha_2 \neq \sigma_1^2$, and is solution of (6), that contradicts (a).

$(b) \Leftrightarrow (c)$ since both (b) and (c) means that $\widetilde{X}_{11}$ and $\widetilde{X}_{22}$ are not colinear.

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